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Adaptive Output Feedback Tracking Control for Nonlinear Systems with Unknown Growth Rate

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Abstract: In this paper, the problem of adaptive output feedback tracking is considered for a class of nonlinear systems with lower-triangular structures. A novel dynamic gain is introduced to deal with the unknown growth rate. By coupling the dynamic gain with the observer and the controller, an adaptive output tracking controller is developed, which can guarantee that all signals of the closed-loop system are globally bounded. Finally, the effectiveness of the presented control scheme is illustrated by a numerical example.

Keywords: nonlinear systems; output feedback; tracking control; unknown growth rate; dynamic gain

1. Introduction

This paper considers the following nonlinear systems:

$$\begin{aligned} &\dot{z}_i = z_{i+1} + f_i(z),\ i = 1,\cdots,n-1, \\ &\dot{z}_n = v + f_n(z), \\ &y = z_1 - y_r, \end{aligned} \tag{1}$$

where $z = (z_1,\cdots,z_n)^T \in \mathbb{R}^n$, $v \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system state, control input and measured output, respectively. y_r denotes the time-varying reference signal. $f_i(\cdot)$, $i = 1,\cdots,n$, are uncertain continuous nonlinear functions.

Owing to the incomplete measurability of all states in practical systems, the output feedback control method is an important and classical control approach in the control community [1–4]. Over the last decades, great efforts have been made on output feedback control for nonlinear systems [5–7]. Generally speaking, due to the non-separation principle, the output feedback control of nonlinear systems is difficult and challenging. By means of a sampled-data reduced-order observer, the problem of global stabilization was addressed in [6] for nonlinear systems. Under a linear growth condition, the problem of global exponential stabilization in the mean square sense was investigated in [7] for stochastic nonlinear systems by means of memoryless output feedback. In [8], a dual-domination method was presented to design an output feedback controller for nonlinear systems with unknown measurement sensitivity. By utilizing the method of output feedback domination, the considered nonlinear systems in [9] can achieve globally asymptotical stability. A novel sampled-data control scheme was put forward in [10] via the output feedback approach for nonlinear uncertain systems with and without designing any state observer. Note that in the above literature, the nonlinear terms of the studied systems satisfy the linear growth condition. Therefore, a problem naturally arises that when the restriction imposed on the nonlinear terms is relaxed, how to design an output feedback controller such that all states of the studied system are bounded?

As one of the fundamental control issues, the tracking control problem has drawn extensive attention due to its wide applications in dual-arm robots, autonomous surface vessels, motor-drive servo systems, and fixed wing unmanned aerial vehicles. Many excellent results have been reported on dealing with tracking problems of nonlinear systems [11–15]. In [12], by using a modified high-gain observer, a tracking control scheme was designed for a class of nonlinear systems subject to unknown parameters. By combining a neural network observer with the adaptive dynamic programming technique, the optimal tracking control algorithm was developed in [13] for continuous-time nonlinear systems. A novel formulation of the time-varying tracking control problem of high-order nonlinear systems was put forward in [14] with time-varying asymmetric output constraints. The authors of [16] investigated the problem of global practical output tracking for nonlinear systems with lower-triangular forms. In [17], the problem of global practical tracking control was addressed by virtue of the dynamic high-gain scaling method.

Inspired by the aforementioned analysis, the objective of this paper is to design a tracking control scheme for nonlinear systems with unknown growth rates. The main contributions of this paper can be summarized as the following three aspects. 1) The output tracking control problem of nonlinear systems is studied by utilizing the power integrator technique. 2) Compared with [8–10, 16], the nonlinear systems under consideration are more general as the included nonlinearities satisfy the homogeneous growth condition and have unknown growth rates. 3) A novel dynamic gain is designed to handle the unknown growth rate.

2. Preliminaries

We use the following assumptions and lemmas in analysis.

Assumption 1. For each $i = 1,\cdots,n$, there exists an unknown constant $c > 0$ such that

$$|f_i(\cdot)| \leqslant c\left(|z_1|^{\frac{r_i+\tau}{r_1}} + |z_2|^{\frac{r_i+\tau}{r_2}} + \cdots + |z_i|^{\frac{r_i+\tau}{r_i}}\right), \tag{2}$$

where $\tau \geqslant 0$ and r_i is defined by

$$r_1 = 1,\ r_{i+1} = r_i + \tau,\ i = 1,\cdots,n. \tag{3}$$

Assumption 2. The reference signal y_r and its first-order derivative $\dot{y}_r$ are bounded.

Remark 1. It is worth noting that many systems may meet Assumption 1 in practice, such as the robotic systems and chemical systems. Compared with [8–10, 16], the growth condition given in Assumption 1 is weaker. Specially, when c is a known constant and $\tau = 0$, the growth condition reduces to the linear growth condition in [8, 9]; and when c is a known constant, the growth condition is equivalent to the condition given by Assumption 3.1 of [16].

Remark 2. Different from [15], Assumption 2 only requires the boundedness of y_r and $\dot{y}_r$, and is independent of the high-order derivatives of y_r.

Lemma 1. [18] For $c \in \mathbb{R}$ and $d \in \mathbb{R}$, if $\hbar_0 \geqslant 1$, one has that

(i) $|c+d|^{\hbar_0} \leqslant 2^{\hbar_0 - 1}\left|c^{\hbar_0} + d^{\hbar_0}\right|$,

(ii) $(|c|+|d|)^{\frac{1}{\hbar_0}} \leqslant |c|^{\frac{1}{\hbar_0}} + |d|^{\frac{1}{\hbar_0}} \leqslant 2^{\frac{\hbar_0 - 1}{\hbar_0}}(|c|+|d|)^{\frac{1}{\hbar_0}}$.

If $\hbar_0 \geqslant 1$ and $\hbar_0$ is a ratio of odd integers, one has that

(iii) $\left|c^{\frac{1}{\hbar_0}} - d^{\frac{1}{r}}\right| \leqslant 2^{1-\frac{1}{\hbar_0}}|c-d|^{\frac{1}{\hbar_0}}$,

(iv) $|c-d|^{\hbar_0} \leqslant 2^{\hbar_0 - 1}\left|c^{\hbar_0} - d^{\hbar_0}\right|$.

Lemma 2. [19] Assume $V: \mathbb{R}^n \to \mathbb{R}$ is homogeneous of the degree (HOD) τ with respect to the dilation Δ. Then, we have that

(i) $\dfrac{\partial V}{\partial x_i}$ is HOD $\tau - r_i$;

(ii) for a constant $\bar{\ell}_0 > 0$, one has $V(x) \leqslant \bar{\ell}_0 \|x\|_\Delta^\tau$. Assume that $V(x)$ is positive definite, then for $\underline{\ell}_0 > 0$, one has $\underline{\ell}_0 \|x\|_\Delta^\tau \leqslant V(x)$.

Lemma 3. [18] For any $\tau \geqslant 0$, there exist constants $a_i > 0$, $i = 1,\cdots,n$, such that the following system:

$$\begin{aligned} \dot{e}_i &= e_{i+1} - a_i e_1^{r_{i+1}},\ i = 1,\cdots,n-1 \\ \dot{e}_n &= -a_n e_1^{r_{n+1}} \end{aligned} \tag{4}$$

is globally asymptotically stable where r_i is the homogenous weight of e_i defined as

$$r_1 = 1,\ r_{i+1}p_i = r_i + \tau,\ i = 1,\cdots,n. \tag{5}$$

3. Main Results

3.1. Observer Design

For system (1), we design the following observer:

$$\begin{aligned} \dot{\hat{z}}_i &= \hat{z}_{i+1} + a_i L^i (y - \hat{z}_1)^{r_{i+1}},\ i = 1,\cdots,n-1 \\ \dot{\hat{z}}_n &= v + a_i L^n (y - \hat{z}_1)^{r_{n+1}} \end{aligned} \tag{6}$$

where $\hat{z} = (\hat{z}_1,\cdots,\hat{z}_n)^T$, constants $a_i > 0$, $i = 1,\cdots,n$, are chosen according to [18], and L is a dynamic gain determined later. Then, a set of scaling-gain changes is introduced as follows:

$$x_1 = z_1 - y_r,\ x_i = \frac{z_i}{L^{i-1}},\ \hat{x}_i = \frac{\hat{z}_i}{L^{i-1}},\ u = \frac{v}{L^n},\ i = 1,\cdots,n, \tag{7}$$

under which, system (1) becomes

$$\begin{aligned}\dot{x}_i &= Lx_{i+1} - (i-1)\frac{\dot{L}}{L}x_i + \varphi_i,\ i = 1,\cdots,n-1,\\ \dot{x}_n &= Lu - (n-1)\frac{\dot{L}}{L}x_n + \varphi_n,\end{aligned} \tag{8}$$

where $\varphi_1 = f_1 - \dot{y}_r$, $\varphi_i = \frac{f_i}{L^{i-1}}$ and $i = 2,\cdots,n$. Under the coordinate change (6) and Assumptions 1-2, there exist positive constants $\bar{c}_1$ and $\tilde{c}_0$ such that

$$\begin{aligned}\varphi_i &\leqslant \frac{c}{L^{i-1}}\left(|z_1|^{\frac{r_i+\tau}{r_1}} + \cdots + |z_i|^{\frac{r_i+\tau}{r_i}}\right) + \frac{|\dot{y}_r|}{L^{i-1}}\\ &\leqslant \frac{c}{L^{i-1}}\left(|x_1 + y_r|^{\frac{r_i+\tau}{r_1}} + \cdots + |L^{i-1}x_i|^{\frac{r_i+\tau}{r_i}}\right) + \frac{|\dot{y}_r|}{L^{i-1}}\\ &\leqslant \bar{c}_1 L^{1-\alpha}\left(|x_1|^{\frac{r_i+\tau}{r_1}} + \cdots + |x_i|^{\frac{r_i+\tau}{r_i}}\right) + \frac{|\dot{y}_r| + \bar{c}_1|y_r|^{r_i+\tau}}{L^{i-1}}\\ &\leqslant \bar{c}_1 L^{1-\alpha}\left(|x_1|^{\frac{r_i+\tau}{r_1}} + \cdots + |x_i|^{\frac{r_i+\tau}{r_i}}\right) + \frac{\tilde{c}_0}{L^{i-1}},\end{aligned} \tag{9}$$

where $\alpha = \frac{1}{r_n}$.

The above equation together with (6) and (7) leads to

$$\begin{aligned}\dot{\hat{x}}_i &= L(\hat{x}_{i+1} + a_i(x_1 - \hat{x}_1)^{r_{i+1}}) - (i-1)\frac{\dot{L}}{L}\hat{x}_i,\ i = 1,\cdots,n-1,\\ \dot{\hat{x}}_n &= L(u + a_n(x_1 - \hat{x}_1)^{r_{n+1}}) - (n-1)\frac{\dot{L}}{L}\hat{x}_n.\end{aligned} \tag{10}$$

Defining the observer error by $e_i = x_i - \hat{x}_i$, one has that

$$\begin{aligned}\dot{e}_i &= L(e_{i+1} - a_i\hat{e}_1^{r_{i+1}}) - (i-1)\frac{\dot{L}}{L}e_i + \varphi_i,\ i = 1,\cdots,n-1,\\ \dot{e}_n &= -La_n\hat{e}_1^{r_{n+1}} - (n-1)\frac{\dot{L}}{L}e_n + \varphi_n.\end{aligned} \tag{11}$$

The following proposition is introduced, which can be obtained from [18] and [20].

Proposition 1. There exists a Lyapunov function V_e with an HOD $\mu = 2r_n$ such that

$$\dot{V}_e \leqslant -L\sum_{i=1}^{n}|e_i|^{\frac{\mu+\tau}{r_i}} - \frac{\dot{L}}{L}\sum_{i=1}^{n}(i-1)\frac{\partial V_e}{\partial e_i}e_i + \sum_{i=1}^{n}\frac{\partial V_e}{\partial e_i}\varphi_i, \tag{12}$$

where $e = (e_1,\cdots,e_n)^T$.

According to Lemma 2, there is a constant $d_1 > 0$ such that

$$\left|\frac{\partial V_e}{\partial e_i}\right| \leqslant d_1|e_1|^{\frac{\mu-r_i}{r_1}} + \cdots + |e_n|^{\frac{\mu-r_i}{r_n}}). \tag{13}$$

By weighted homogeneity, one has

$$\sum_{i=1}^{n}|e_i|^{\frac{\mu+\tau}{r_i}} \geqslant g_1\|e\|_{\Delta}^{\mu+\tau}, \tag{14}$$

where g_1 is a positive constant.

Define the following adaptive law:

$$\dot{L} = \lambda L^2 \max\{L^{-\alpha}(y - y_r - \hat{z}_1)^{\tau} - \omega, 0\},\ L(0) = 1, \tag{15}$$

where λ and ω are two positive constants.

It can be deduced from Lemma 1 and (13) that

$$-\frac{\dot{L}}{L}\sum_{i=1}^{n}(i-1)\frac{\partial V_e}{\partial e_i}e_i \leqslant d_1\frac{\dot{L}}{L}\sum_{i=1}^{n}(i-1)\|e\|_\Delta^{\mu-r_i}\|e\|_\Delta^{r_i}$$
$$=\frac{d_1}{2}n(n-1)\frac{\dot{L}}{L}\|e\|_\Delta^{\mu}$$
$$\leqslant \rho_0 L^{1-\alpha}\|e\|_\Delta^{\mu+\tau}, \tag{16}$$

where ρ_0 is a positive constant.

By Lemma 1 and Young's inequality, one has that

$$\sum_{i=1}^{n}\frac{\partial V_e}{\partial e_i}\varphi_i \leqslant d_1\bar{c}_1 L^{1-\alpha}\sum_{i=1}^{n}\left(\|e\|_\Delta^{\mu-r_i}\left(|x_1|^{\frac{r_i+\tau}{r_1}}+\cdots+|x_i|^{\frac{r_i+\tau}{r_i}}\right)\right)+\tilde{c}_0 d_1\sum_{i=1}^{n}\left(\|e\|_\Delta^{\mu-r_i}\frac{1}{L^{i-1}}\right)$$
$$\leqslant d_1\bar{c}_1 L^{1-\alpha}\sum_{i=1}^{n}\left(\|e\|_\Delta^{\mu-r_i}\left(|e_1+\hat{x}_1|^{\frac{r_i+\tau}{r_1}}+\cdots+|e_i+\hat{x}_i|^{\frac{r_i+\tau}{r_i}}\right)\right)+\tilde{c}_0 d_1\sum_{i=1}^{n}\left(\|e\|_\Delta^{\mu-r_i}\frac{1}{L^{i-1}}\right)$$
$$\leqslant c_1 L^{1-\alpha}\left(\|e\|_\Delta^{\mu+\tau}+\|\hat{x}\|_\Delta^{\mu+\tau}\right)+\tilde{c}_0 d_1\sum_{i=1}^{n}\left(\|e\|_\Delta^{\mu-r_i}\frac{1}{L^{i-1}}\right), \tag{17}$$

where $c_1>0$ and $\tilde{c}_0>0$ are constants.

By virtue of Young's inequality, there exists a positive constant $\hat{c}_0$ such that

$$\tilde{c}_0 d_1\|e\|_\Delta^{\mu-r_i}\frac{1}{L^{i-1}} \leqslant \tilde{c}_0 d_1\|e\|_\Delta^{\mu-r_i}\left(L^{-\frac{(i-1)}{r_i+\tau}}\right)^{r_i+\tau}$$
$$\leqslant \frac{g_1}{2n}\|e\|_\Delta^{\mu+\tau}+\hat{c}_0 L^{-\frac{(i-1)(\mu+\tau)}{r_i+\tau}}, \tag{18}$$

which together with (16) and (17) indicates

$$\dot{V}_e \leqslant -L\sum_{i=1}^{n}|e_i|^{\mu+\tau}+c_1 L^{1-\alpha}(\|e\|_\Delta^{\mu+\tau}+\|\hat{x}\|_\Delta^{\mu+\tau})$$
$$+\rho_0 L^{1-\alpha}\|e\|_\Delta^{\mu+\tau}+\frac{g_1}{2}\|e\|_\Delta^{\mu+\tau}+\hat{c}_0\sum_{i=1}^{n}L^{-\frac{(i-1)(\mu+\tau)}{r_i+\tau}}. \tag{19}$$

By (14), it can be seen that $L\geqslant 1$. Then, (19) becomes

$$\dot{V}_e \leqslant -\frac{g_1}{2}L\|e\|_\Delta^{\mu+\tau}+\check{c}_1 L^{1-\alpha}(\|e\|_\Delta^{\mu+\tau}+\|\hat{x}\|_\Delta^{\mu+\tau})+\hat{c}_0\sum_{i=1}^{n}L^{-\frac{(i-1)(\mu+\tau)}{r_i+\tau}}, \tag{20}$$

where $\check{c}_1=c_1+\rho_0$.

3.2. Controller Design

Choose the following Lyapunov function:

$$V_1=\int_{\hat{x}_1^*}^{\hat{x}_1}\left(s^{\frac{r_{n+1}}{r_1}}-\hat{x}_1^{*\frac{r_{n+1}}{r_1}}\right)^{\frac{\mu-r_1}{r_{n+1}}}ds, \tag{21}$$

where $\hat{x}_1^*=0$. Its derivative is

$$\dot{V}_1=L\hat{x}_1^{\frac{\mu-r_1}{r_1}}\left(\hat{x}_2+a_1e_1^{r_2}\right). \tag{22}$$

It follows from Young's inequality that

$$L\hat{x}_1^{\frac{\mu-r_1}{r_1}}a_1e_1^{r_2} \leqslant \rho_1 L|\hat{x}_1|^{\frac{\mu+\tau}{r_1}}+\tfrac{g_1}{4n}L|e_1|^{\mu+\tau}, \tag{23}$$

where $\rho_1=(\mu-r_1)a_1^{\frac{\mu+\tau}{\mu-r_1}}(4nr_2)^{\frac{r_2}{\mu-r_1}}/\left((\mu+\tau)^{\frac{\mu+\tau}{\mu-r_1}}g_1^{\frac{r_2}{\mu-r_1}}\right)$.

The virtual controller $\hat{x}_2^*$ is defined as

$$\hat{x}_2^*=-\beta_1\xi_1^{\frac{r_2}{r_{n+1}}},\ \beta_1=(n+\rho_1), \tag{24}$$

where $\xi_1=\hat{x}_1^{r_{n+1}/r_1}$. Then, it is not difficult to deduce that

$$\dot{V}_1 \leqslant -nL|\xi_1|^{\frac{\mu+\tau}{r_{n+1}}}+\tfrac{g_1}{4n}L|e_1|^{\mu+\tau}+L\xi_1^{\frac{\mu-r_1}{r_{n+1}}}\left(\hat{x}_2-x_2^*\right). \tag{25}$$

Construct the following Lyapunov function:

$$V_2 = V_1 + W_2, \tag{26}$$

where $W_2 = \int_{\hat{x}_2^*}^{\hat{x}_2} \left(s^{\frac{r_{n+1}}{r_2}} - \hat{x}_2^{*\frac{r_{n+1}}{r_2}} \right)^{\frac{\mu - r_2}{r_{n+1}}} ds$. Then, the time derivative of V_2 is calculated by

$$\begin{aligned}
\dot{V}_2 =& \dot{V}_1 + \xi_2^{\frac{\mu-r_2}{r_{n+1}}} \left(L(\hat{x}_3 + a_2 e_1^{r_3}) - \frac{\dot{L}}{L}\hat{x}_2 \right) + \frac{\partial W_2}{\partial \hat{x}_1} L(\hat{x}_2 + a_1 e_1^{r_2}) \\
\leqslant& -nL|\xi_1|^{\frac{\mu+\tau}{r_{n+1}}} + \frac{g_1}{4n} L|e_1|^{\mu+\tau} + L\xi_1^{\frac{\mu-r_1}{r_{n+1}}} \left(\hat{x}_2 - x_2^* \right) + \xi_2^{\frac{\mu-r_2}{r_{n+1}}} \left(L(\hat{x}_3 + a_2 e_1^{r_3}) - \frac{\dot{L}}{L}\hat{x}_2 \right) \\
& - \frac{\mu - r_2}{r_{n+1}} \int_{\hat{x}_2^*}^{\hat{x}_2} \left(s^{\frac{r_{n+1}}{r_2}} - x_2^{*\frac{r_{n+1}}{r_2}} \right)^{\frac{\mu-r_2}{r_{n+1}} - 1} ds \frac{\partial \hat{x}_2^{*\frac{r_{n+1}}{r_2}}}{\partial \hat{x}_1} L(\hat{x}_2 + a_1 e_1^{r_2}),
\end{aligned} \tag{27}$$

where $\xi_2 = \hat{x}_2^{\frac{r_{n+1}}{r_2}} - \hat{x}_2^{*\frac{r_{n+1}}{r_2}}$.

By using Young's inequality, it can be deduced that

$$L\xi_1^{\frac{\mu-r_1}{r_{n+1}}} \left(\hat{x}_2 - x_2^* \right) \leqslant \frac{L}{3} |\xi_1|^{\frac{\mu+\tau}{r_{n+1}}} + \rho_{21} L |\xi_2|^{\frac{\mu+\tau}{r_{n+1}}}, \tag{28}$$

$$L\xi_1^{\frac{\mu-r_2}{r_{n+1}}} a_2 e_1^{r_3} \leqslant \rho_{22} L |\xi_2|^{\frac{\mu+\tau}{r_{n+1}}} + \tfrac{g_1}{12n} L|e_1|^{\mu+\tau}, \tag{29}$$

where $\rho_{21} = r_2 2^{\frac{\mu+\tau}{r_2} - \frac{\mu+\tau}{r_{n+1}}} 3^{\frac{\mu-r_1}{r_2}} (\mu - r_1)^{\frac{\mu-r_1}{r_2}} / (\mu+\tau)^{\frac{\mu+\tau}{r_2}}$ and $\rho_{22} = (\mu - r_2)(12nr_3)^{\frac{r_3}{\mu-r_2}} a_2^{\frac{\mu+\tau}{\mu-r_2}} / \left(g_1^{\frac{r_3}{\mu-r_2}} (\mu+\tau)^{\frac{\mu+\tau}{\mu-r_2}} \right)$.

Similarly, we have that

$$\begin{aligned}
& -\frac{\mu - r_2}{r_{n+1}} \int_{\hat{x}_2^*}^{\hat{x}_2} \left(s^{\frac{r_{n+1}}{r_2}} - x_2^{*\frac{r_{n+1}}{r_2}} \right)^{\frac{\mu-r_2}{r_{n+1}} - 1} ds \frac{\partial \hat{x}_2^{*\frac{r_{n+1}}{r_2}}}{\partial \hat{x}_1} L(\hat{x}_2 + a_1 e_1^{r_2}) \\
& \leqslant \frac{\mu - r_2}{r_{n+1}} |\xi_2|^{\frac{\mu - r_{n+1}}{r_{n+1}}} |\hat{x}_2 - \hat{x}_2^*| \left| \frac{\partial \hat{x}_2^{*\frac{r_{n+1}}{r_2}}}{\partial \hat{x}_1} \right| |L(\hat{x}_2 + a_1 e_1^{r_2})| \\
& \leqslant \frac{\mu - r_2}{r_{n+1}} 2^{1 - \frac{r_2}{r_{n+1}}} |\xi_2|^{\frac{\mu+r_2}{r_{n+1}} - 1} \beta_1^{\frac{r_{n+1}}{r_2}} \frac{r_{n+1}}{r_1} |\xi_1|^{1 - \frac{r_1}{r_{n+1}}} L \left(|\xi_2 - \beta_1^{\frac{r_{n+1}}{r_2}} \xi_1|^{\frac{r_2}{r_{n+1}}} + a_1 |e_1|^{r_2} \right) \\
& \leqslant \frac{L}{3} |\xi_1|^{\frac{\mu+\tau}{r_{n+1}}} + \rho_{23} L |\xi_2|^{\frac{\mu+\tau}{r_{n+1}}} + \frac{g_1}{12n} L|e_1|^{\mu+\tau},
\end{aligned} \tag{30}$$

$$\begin{aligned}
-\frac{\dot{L}}{L} \xi_1^{\frac{\mu-r_2}{r_{n+1}}} \hat{x}_2 \leqslant& \lambda L^{1-\alpha} |e_1|^{\tau} |\xi_1|^{\frac{\mu-r_2}{r_{n+1}}} |\xi_2 - \beta_1^{\frac{r_{n+1}}{r_2}} \xi_1|^{\frac{r_2}{r_{n+1}}} \\
\leqslant& \frac{1}{3} L |\xi_1|^{\frac{\mu+\tau}{r_{n+1}}} + \rho_{24} L |\xi_2|^{\frac{\mu+\tau}{r_{n+1}}} + \frac{g_1}{12n} L|e_1|^{\mu+\tau},
\end{aligned} \tag{31}$$

where ρ_{23} and ρ_{24} are two positive constants. Then, one can choose the virtual controller $\hat{x}_3^*$ as

$$\hat{x}_3^* = -\beta_2 \xi_2^{\frac{r_3}{r_{n+1}}}, \ \beta_2 = n - 1 + \rho_{21} + \rho_{22} + \rho_{23} + \rho_{24}, \tag{32}$$

which guarantees

$$\dot{V}_2 \leqslant -(n-1)L \sum_{j=1}^{2} |\xi_j|^{\frac{\mu+\tau}{r_{n+1}}} + \frac{g_1}{2n} L|e_1|^{\mu+\tau} + \xi_2^{\frac{\mu-r_2}{r_{n+1}}} L(\hat{x}_3 - \hat{x}_3^*). \tag{33}$$

Suppose that at step i, there is a Lyapunov function V_i and a set of virtual controllers $\hat{x}_1^*, \ldots, \hat{x}_i^*$ defined by

$$\begin{aligned}
\hat{x}_1^* &= 0, & \xi_1 &= \hat{x}_1^{\frac{r_{n+1}}{r_1}} - \hat{x}_1^{*\frac{r_{n+1}}{r_1}} \\
\hat{x}_j^* &= -\beta_{j-1} \xi_{j-1}^{\frac{r_j}{r_{n+1}}}, & \xi_j &= \hat{x}_j^{\frac{r_{n+1}}{r_j}} - \hat{x}_j^{*\frac{r_{n+1}}{r_j}}, j = 2, \cdots, i
\end{aligned} \tag{34}$$

such that

$$\dot{V}_i \leqslant -(n-i+1)L \sum_{j=1}^{i} |\xi_j|^{\frac{\mu+\tau}{r_{n+1}}} + \frac{i}{4n} g_1 L|e_1|^{\mu+\tau} + \xi_i^{\frac{\mu-r_i}{r_{n+1}}} L(\hat{x}_{i+1} - \hat{x}_{i+1}^*), \tag{35}$$

where β_j are positive constants.

Finally, at step n, there exists a Lyapunov function

$$V_n=\sum_{i=1}^{n}\int_{\hat{x}_i^*}^{\hat{x}_i}\left(s^{\frac{r_{n+1}}{r_i}}-\hat{x}_i^{*\frac{r_{n+1}}{r_i}}\right)^{\frac{\mu-r_i}{r_{n+1}}}ds, \tag{36}$$

such that

$$\dot{V}_n\leqslant -L\sum_{i=1}^{n}|\xi_i|^{\frac{\mu+\tau}{r_{n+1}}}+\frac{g_1}{4}L|e_1|^{\mu+\tau}+L\xi_n^{\frac{\mu-r_n}{r_{n+1}}}(u-\hat{x}_{n+1}^*). \tag{37}$$

Then, the controller is constructed as

$$u=\hat{x}_{n+1}^*. \tag{38}$$

By weighted homogeneity, it can be inferred that

$$\sum_{i=1}^{n}|\xi_i|^{\frac{\mu+\tau}{r_{n+1}}}\geqslant g_2\|\hat{x}\|_\Delta^{\mu+\tau}, \tag{39}$$

where g_2 is a positive constant. Then, substituting (38) and (39) into (37) yields

$$\dot{V}_n\leqslant -g_2L\|\hat{x}\|_\Delta^{\mu+\tau}+\tfrac{g_1}{4}L|e_1|^{\mu+\tau}. \tag{40}$$

Theorem 1. For system (1) satisfying Assumptions 1-2, the problem of global practical tracking can be solved by the adaptive output feedback controller comprised of (6), (15) and (38), and all signals of the closed-loop system are bounded.

Proof. Taking into account $|e_1|\leqslant\|e\|_\Delta$ and combining (20) and (40), we have

$$\dot{V}\leqslant -L\left(\frac{g_1}{4}-\check{c}_1L^{-\alpha}\right)\|e\|_\Delta^{\mu+\tau}-L(g_2-\check{c}_1L^{-\alpha})\|\hat{x}\|_\Delta^{\mu+\tau}+\gamma_1, \tag{41}$$

where $V=V_e+V_n$ and $\gamma_1=n\hat{c}_0$.

From the construction of $\dot{L}$, we have $\dot{L}\geqslant 0$ and $L(t)\geqslant L(0)=1$. For any initial value $(z(0),\hat{z}(0),L(0))$, the resultant closed-loop system has a unique solution on the maximal existence interval $[0,t_f)$. Then, we will prove that $(z(t),\hat{z}(t),L(t))$ is bounded.

Assume that $L(t)$ cannot escape at $t=t_f$. Due to $\dot{L}(t)\geqslant 0$, $L(t)$ is a monotonically nondecreasing function. Hence, there exists a time t_0 such that

$$L(t)\geqslant\max\{\left(\frac{8\check{c}_1}{g_1}\right)^{\frac{1}{\alpha}},\left(\frac{2\check{c}_1}{g_2}\right)^{\frac{1}{\alpha}}\}\},\forall t\in[t_0,t_f) \tag{42}$$

which together with (41) indicates

$$\dot{V}\leqslant -\frac{g_1}{8}L\|e\|_\Delta^{\mu+\tau}-\frac{g_2}{2}L\|\hat{x}\|_\Delta^{\mu+\tau}+\gamma_1. \tag{43}$$

Thus, it is concluded that $(x(t),\hat{x}(t))$ is bounded on $[0,t_f)$. Suppose $\lim_{t\to t_f}L(t)=+\infty$. Recalling (15), one has $\dot{L}=0$, which contradicts the hypothesis that $\lim_{t\to t_f}L(t)=+\infty$. Hence, $L(t)$ is bounded on $[0,t_f)$. Based on (7), it is concluded that $(z(t),\hat{z}(t))$ is bounded on $[0,t_f)$. From the continuity of the solution, it can be deduced that $t_f=+\infty$. This implies that $(z(t),\hat{z}(t),L(t))$ is bounded on $[0,+\infty)$.

Remark 3. As can be seen from Assumption 1, the growth rate c is unknown and its information cannot be directly utilized in controller design. This makes the controller design more difficult. As a consequence, a dynamic gain L is introduced to cope with the unknown growth rate.

Remark 4. In the controller design process, we have adopted the technique of adding a power integrator, which utilizes the homogeneous negative term $-|\xi_1|^{\frac{\mu+\tau}{r_{n+1}}}$ in (25) to dominate the superfluous term in the subsequent step.

4. Simulation Example

To illustrate the effectiveness of the proposed control approach, a numerical example is provided in this section.

Example 1. Consider the following system:

$$\begin{aligned}\dot{z}_1&=z_2+cz_1^{\frac{11}{9}},\\ \dot{z}_2&=v+cz_2^{\frac{13}{11}},\\ y&=z_1-y_r,\end{aligned} \tag{44}$$

where c is an unknown constant. It is easy to see that Assumption 1 holds with $\tau = \frac{2}{9}$.

The control aim is to track the reference signal $y_r = \sin(t)$. In the simulation, the parameters are set as $c = 0.1$, $a_1 = 18$, $a_2 = 38$, $\beta_1 = 29$, $\beta_2 = 49$, $\lambda = 1$, $\alpha = \frac{2}{9}$ and $\omega = 0.55$. The initial conditions are $z_1(0) = 0.5$, $z_2(0) = -2$, $\hat{z}_1(0) = 0$ and $\hat{z}_2(0) = 0$. Simulation results are shown in Figures 1−3, from which it can be seen that all closed-loop signals are bounded. To illustrate the benefit of our controller, a comparative simulation is conducted between our control scheme and the control scheme in [16] with the same parameters. Figure 1 shows the response curves of the system state z_1 and the reference signal y_r. Clearly, our control scheme achieves better tracking performance than the scheme given by [16].

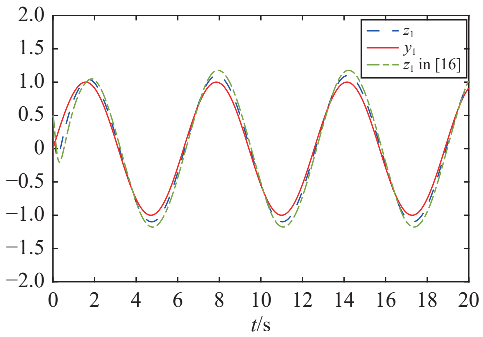

Figure 1. The curves of the system state z_1 and the reference signal y_r.

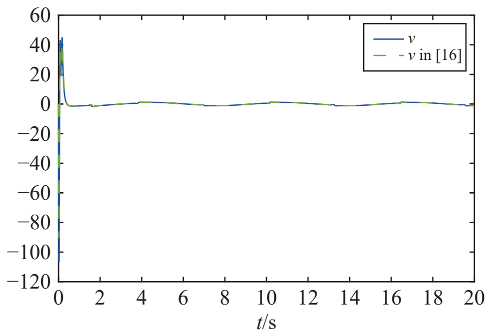

Figure 2. The curve of the control input v.

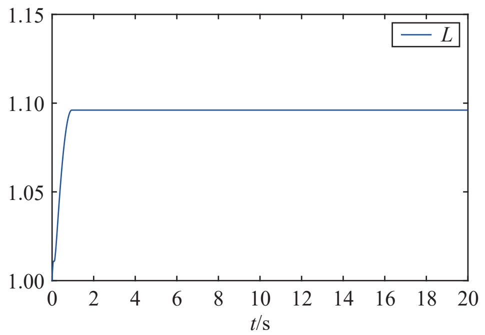

Figure 3. The curve of the dynamic gain L.

5. Conclusion

For a class of nonlinear systems with unknown growth rates, the tracking control problem has been solved via the output feedback strategy. A novel dynamic gain is introduced to eliminate the effect of the unknown growth rate. A dynamic-gain observer and an adaptive output feedback controller has been developed by means of adding a power integrator technique. A numerical example has been given to verify the validity of the presented result. Inspired by the current work, future work will focus on the prescribed-time fault-tolerant control of uncertain nonlinear systems.

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